

Shot noise in long conductors

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Shot noise is the only fundamental type of current fluctuations which exists in current-carrying conductors at low temperatures. As was recently stressed by Landauer [1], it contains unique and important information about the correlations which affect the electronic transport in the conductor. It is generally accepted that shot noise is a 'mesoscopic' phenomenon in the sense that it vanishes when the length of the conductor is much larger than the electron-phonon thermalization length [2]. We show here that at least in one geometry this result is valid only when the observation frequency ω is strictly zero. At any finite frequency the shot noise, as measured in the electrodes connecting the conductor, resumes its value of the order of the 'full' shot noise $2eI$ (I is the average current in the conductor) provided that the sample is *long* enough, $L > L_0(\omega)$. Since even 'zero frequency' measurements of shot noise are always done at RF, we claim here that for sufficiently long samples (typically longer than a few centimeters) such measurements should yield a large shot noise value.

The basis for the calculations leading to the above conclusion is the 'drift-diffusion-Langevin' theory developed in Refs. [3,4] and valid at $\omega \ll 1/\tau$, eV/\hbar with τ the elastic scattering time and V the applied voltage. According to this theory the noise spectral density as measured in the electrodes connecting the conductor (of length L centered around $x = 0$) can be presented as

$$S_I(\omega) = \frac{2G}{L} \int_{-\frac{L}{2}}^{\frac{L}{2}} |K(x; \omega)|^2 \mathcal{C}(x) dx \quad (1)$$

with G the dc conductance and $\mathcal{C}(x)$ the correlator of local fluctuations. This correlator depends only on the local distribution function of electrons. For example, for an equilibrium Fermi-Dirac distribution with temperature T $\mathcal{C}(x) = 2T$, and the noise assumes the Johnson-Nyquist value. For typical non-equilibrium distributions $\mathcal{C}(x)$ becomes much larger than the equilibrium correlator, and at $T = 0$ the noise given by Eq. (1) is the shot noise.

The response function $K(x; \omega)$, which is solely responsible for the frequency dispersion of the noise, gives the current generated in the electrodes by a fluctuating unit current source at x . It is dependent upon the specific geometry of the conductor and its electrodynamic environment, but its integral over the sample length always equals 1. We will study here a simple geometry

for which an analytical expression for $K(x; \omega)$ can be obtained, namely, a thin and long conductor close to a ground plane. It is assumed that both thickness of the conductor (in the direction perpendicular to the ground plane) and its distance from the ground plane are much smaller than L [5]. Then the response function is given by [4]

$$K(x; \omega) = \kappa \frac{L}{2} \frac{\cosh(\kappa x)}{\sinh(\kappa L/2)}. \quad (2)$$

Here $\kappa(\omega) = \sqrt{-i\omega/D'}$ with $D' = D + GL/C_0$ where D is the diffusion coefficient and C_0 is the (dimensionless) linear capacitance between the conductor and the ground plane.

It is clear from Eq. (2) that the only current fluctuations in the conductor which are of importance in inducing noise in the electrodes are those which are within a distance $\lambda_\omega = 1/|\kappa(\omega)|$ from the conductor-electrode interfaces. Therefore, at high enough frequencies, the measured noise is associated with the highly non-equilibrium distribution of electrons near the edges of the conductor, and not necessarily with the distribution at the bulk of the sample (in long samples, the latter can be very close to an equilibrium distribution, a fact which led to the common belief that the shot noise should vanish in long samples). This simple argument means that whenever λ_ω is smaller than some length scale l_S (which gives the spatial extent of non-equilibrium electrons in the conductor – see Fig. 1), the shot noise value should remain significant even with increasing L .

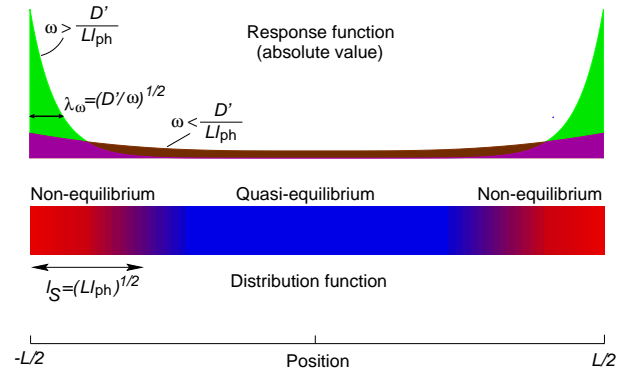


FIG. 1. Schematic description of the relevant length scales in the problem: The upper curve shows the response function for the two cases of relatively low (brown) and high (green) frequencies. The lower panel shows the spatial extent of non-equilibrium electrons near the edges of the conductor.

In order to give a quantitative description of the above effect, one should solve the Boltzmann equation for the non-equilibrium distribution f . To this end, we follow the prescription of Ref. [6], where f was calculated under

the assumption of longitudinal acoustic phonon scattering. It was shown in that work that the width of the layer in which the electron distribution is far from equilibrium is $l_S \approx \sqrt{L l_{\text{ph}}}$, with l_{ph} the inelastic scattering length of an electron due to emission of a phonon of energy eV . Therefore, one should expect large shot noise if $\lambda_\omega \approx \sqrt{D'/\omega} < \sqrt{L l_{\text{ph}}}$, or $L > L_0(\omega)$ with

$$L_0(\omega) = \frac{D'}{l_{\text{ph}}\omega}. \quad (3)$$

In what follows we would be interested in relatively long samples and low frequencies. Therefore we assume here that the electron-electron scattering length is much shorter than L and λ_ω . Having numerical results for f (and thus for \mathcal{C}) in this situation [6], we can find the noise spectral density by combining Equations (1) and (2).

Results for the noise spectral density $S_I(\omega)$ are presented in Fig. 2 for a specific set of experimental parameters. The upper curves in the figure show the total noise. The lower curves show, on the same scale, the thermal noise. Since the latter is smaller by an order of magnitude than the former, the upper curves actually depict the shot noise.

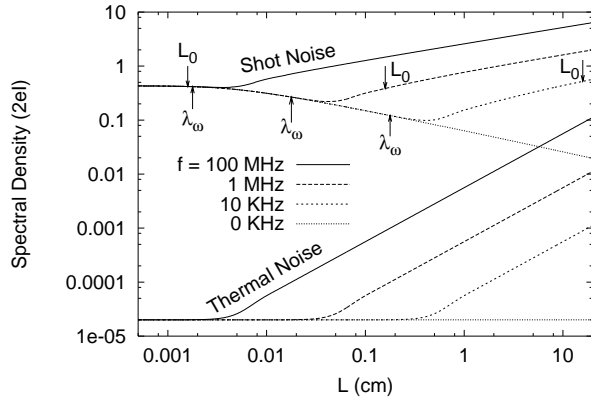


FIG. 2. Noise spectral density as a function of sample's length. Lower curves show the thermal noise and upper curves the full noise which is dominated by the shot noise. Material parameters are $l_{\text{ph}} = 10^{-3}$ cm and $D' = 1000$ cm²/s. Temperature-to-voltage ratio is $T/eV = 10^{-5}$. Arrows indicate the positions of λ_ω and L_0 for each of the depicted frequencies (at zero frequency these lengths tend to infinity).

The physical discussion presented above is fully supported by the results shown in Fig. 2. One sees that at each of the three frequencies depicted, the noise initially decreases with L up to $L \approx \lambda_\omega$, whereupon it increases, and reaches its mesoscopic value again at $L \approx L_0(\omega)$. The initial decrease of the noise with increasing L is due to the electrons being increasingly thermalized in the bulk of the sample, while the subsequent increase is due to the widening of the non-equilibrium surface layer as $\sqrt{L l_{\text{ph}}}$, and therefore the increasing distance from equilibrium

of the noise-inducing electrons within the layer of distance λ_ω from the interfaces (see Fig. 1). As expected, at strictly zero frequency the noise reduces monotonically to the thermal value at $L \rightarrow \infty$.

The unusual result of shot noise increasing with increasing sample length is essentially due to a competition between two independent physical processes: screening and equilibration. The importance of screening in affecting shot noise was first discussed by Landauer in qualitative terms [7,8], and was later studied quantitatively in Refs. [3,4]. Its outcome effect is summarized by Eq. (2) and by the upper panel of Fig. 1. Equilibration, on the other hand (lower panel of Fig. 1) is responsible for the surface layers of non-equilibrium electrons. The fact that the width l_S of this layer grows with L is readily understood [6]: since the electron-phonon relaxation time decreases strongly with the energy of the emitted phonon, at large L , when the electric field in the conductor is small, an electron entering the sample from the electrode must diffuse elastically for a long distance before it is able to emit a phonon.

Parameters chosen in obtaining Fig. 2 (except, possibly, the temperature) correspond to typical experimental situations (c.f. Ref. [9]). In fact, many 'zero frequency' experiments are performed at an actual frequency of 10 KHz or higher. A different choice of the effective diffusion coefficient D' would leave the results unaltered only if accompanied by a similar change of the frequency (in other words, the only dependence of S_I on ω and on D' is through ω/D'). While $D = 1000$ cm²/s is easy to achieve experimentally, the electrostatic term in D' , $GL/C_0 \approx D(td/\lambda_0^2)$, dominates if the thickness t of the conductor or its distance d from the ground plane are larger than the static screening length λ_0 . Thus, the most promising geometry for the observation of the results depicted in Fig. 2 is within a thin layer, possibly a two-dimensional electron gas. With thicker conductors, the macroscopic shot noise would be large only at higher frequencies, or longer samples [see Eq. (3)].

In order to keep L_0 reasonably small l_{ph} must be large. To maintain $l_{\text{ph}} = 10^{-3}$ cm, V cannot be larger than 100 mV. It is therefore likely that in an actual experiment T/eV would not be smaller than 10^{-3} . Then, at large L and ω , the thermal noise may be as large as the shot noise, and a subtraction scheme should be deployed in order to extract the shot noise values from the measurements.

The response function (2), and thus the results presented in this work, are not necessarily valid for geometries different from the one studied here. The question of whether any specific geometry exhibits shot noise when the conductor is long enough reduces to the question whether finite-frequency fluctuations in the bulk of the conductor are sufficiently screened as to not induce current in the electrodes. Theoretically, a detailed answer to this question may involve difficult solutions of

the Poisson equation. However, in a charged Fermi system finite-frequency currents are known to be screened beyond some typical length scale λ'_ω which does not depend on L [10]. On the other hand, the 'hot-electron' length scale $l_S = \sqrt{L l_{ph}}$ is independent of the specific geometry. Therefore, it is argued that in sufficiently long samples of an arbitrary geometry l_S is larger than λ'_ω , so the only important sources of noise are from the non-equilibrium regions near the electrodes. Following the physical discussion below Eq. (2), this implies that the qualitative features of the results presented here may be of a general nature.

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